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Question Paper Code : 81626

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2025.

Third Semester

Electrical and Electronics Engineering

MA 3303 — PROBABILITY AND COMPLEX FUNCTIONS

(Regulations 2021)

Time : Three hours

Maximum : 100 marks

(Use of statistical table is permitted)

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. Two random digits are selected using random number generator (all 100 pairs of digits are equally likely). Find the probability of getting, first a 5 and then a digit less than 5.
2. Write memoryless property for an exponential distribution.
3. List any two uses of marginal distributions in joint probability distribution.
4. Does zero correlation coefficient of two random variables, imply independence of those variables? Justify your answer.
5. Is the complex function $f(z) = z^2 + iz$ analytic?
6. What are fixed points in a bilinear transformation?
7. Evaluate $\oint_C z^3 dz$, where C is the unit circle $|z| = 1$.
8. What are poles and singularities in Laurent's series? Give examples.
9. Solve the differential equation $(D^2 + 3D + 2)y = 0$ where $D = \frac{d}{dx}$.
10. Give the general form of homogeneous Euler's equation.

PART B — ($5 \times 16 = 80$ marks)

11. (a) (i) Customers are used to evaluate preliminary product designs. In the past, 95% of highly successful products received good reviews, 60% of moderately successful products received good reviews and 10% of poor products received good reviews. In addition, 40% of products have been highly successful, 35% have been moderately successful and 25% have been poor products. What is the probability that a product attains a good review? If a new design attains a good review, what is the probability that it will be highly successful product? (8)
- (ii) For the case of a thin copper wire, suppose that the number of flaws follows a Poisson distribution with a mean of 2, 3 flaws per millimeter. Determine the probability of
- (1) Exactly 2 flaws in one millimeter
 - (2) 3 or 4 flaws in one millimeter and
 - (3) Atmost 3 flaws in one millimeter. (8)

Or

- (b) (i) Suppose the probability density function of the length of computer cables is $f(x) = \begin{cases} 0.1x, & 1200 \leq x \leq 1210 \\ 0, & \text{otherwise} \end{cases}$. Determine the mean and standard deviation of the cable length. And also if the length specifications are $1195 < x < 1205$ millimeters, what proportion of cables are within specifications? (8)
- (ii) The compressive strength of samples of cement can be modeled by a normal distribution with a mean of 6000 kg/cm^2 and a standard deviation of 100 kg/cm^2 . (8)
- (1) What is the probability that a sample's strength is less than 6250 Kg/cm^2 ?
 - (2) What is the probability that a sample's strength is between 5800 and 5900 Kg/cm^2 ?
 - (3) What strength is exceeded by 95% of the samples?

12. (a) Determine the value of 'c' that makes the function $f(x,y) = cxy$ a joint probability function over the nine points with $x = 1, 2, 3$ and $y = 1, 2, 3$.
- (i) Determine $P(X < 2, Y < 2)$, $P(X > 1, Y \leq 2)$
 - (ii) Determine correlation coefficient of x and y .

Or

- (b) (i) The following data were collected to determine the relationship between pressure and the corresponding scale reading for the purpose of calibration. (8)

Pressure (x) 10 20 30 40 50

Scale reading (y) 13 18 16 15 20

Construct a linear regression equation y on x for the above data and estimate the scale reading when pressure is $x = 35$.

- (ii) Let X and Y are two random variables having the joint density function $f_{XY}(x, y) = \begin{cases} 4xye^{-(x^2+y^2)}, & x, y \geq 0 \\ 0 & \text{otherwise} \end{cases}$. (8)

- (1) Find marginal of x and y .
- (2) Check whether x and y are independent.
- (3) Find condition distribution of x given y .

13. (a) (i) Is the function $u = e^{-2x} \sin 2y$ harmonic? If so, find its harmonic conjugate, and construct an analytic function, $f(z) = u + iv$. (8)
- (ii) Show that constant absolute value of an analytical function is constant. (8)

Or

- (b) (i) Construct the (bilinear transformation) that maps $(0, 2i, -2i)$ from z -plane onto $(-1, 0, \infty)$ in the w -plane. (8)

- (ii) Discuss about the conformal mapping $w = \frac{1}{z}$ and $w = z^2$. (8)

14. (a) (i) Evaluate $\oint_C \frac{(z^4 - 3z^3 - 2z^2 + 1)}{(z-2)^3} dz$, where C is $|z| = 3$. (8)

- (ii) Find all Taylor and Laurent series of $f(z) = \frac{z+3}{(z-1)(z-4)}$ centered at $z_0 = 0$. Also determine the precise regions of convergence in each case. (8)

Or

- (b) (i) State and prove Cauchy's Residue theorem. (8)

- (ii) Use Cauchy's Residue Theorem to evaluate $\oint_C \frac{30z^2 - 23z + 5}{(2z-1)(3z-1)} dz$, where C is $|z| = 1$. (8)

15. (a) (i) Solve the differential equation $y'' + 4y' + 4y = 3\sin x + 4\cos x$, with the initial conditions $y(0) = 1$ and $y'(0) = 0$ where $y' = \frac{dy}{dx}$, $y'' = \frac{d^2y}{dx^2}$. (8)

- (ii) Solve the non-homogeneous linear ODE by variation of parameters $(D^2 - 2D + 1)y = 6x^2e^{-x}$ where $D = \frac{d}{dx}$, $D^2 = \frac{d^2}{dx^2}$. (8)

Or

- (b) (i) Solve the simultaneous differential equations $\frac{dx}{dt} + 2y + \sin t = 0$, $\frac{dy}{dt} - 2x - \cos t = 0$.

- (ii) Solve the differential equation $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 4y = e^{-2x} - x^2$, by method of undetermined coefficients, assuming the trial solution as $y = a_1x^2 + a_2x + a_3 + a_4e^{-x}$.